P516987.PDF [Page: 3 of 39]

DEFENCE R&D DÉFENSE

Monte Carlo Simulation of Crack Growth on Ship Hull Plating

Samon Ando

Defence R&D Canada

Technical Memorandum

DREA TM 2001-166

October 2001

National Defence Défense nationale

Canada

P516987.PDF [Page: 4 of 39]

Copy No: _____

Monte Carlo Simulation of Crack Growth on Ship Hull Plating

Samon Ando

Defence Research Establishment Atlantic

Technical Memorandom

DREA TM 2001-166

October 2001

516987.PDF [Page: 5 of 39]

Author

S. Ando

Samon Ando

Approved by

J.L. Kennedy

Head/Warship Signatures and Safety Section

Approved for release by

K. Foster

Chair, Document Review Committee

© Her Majesty the Queen as represented by the Minister of National Defence, 2001

© Sa majesté la reine, représentée par le ministre de la Défense nationale, 2001

P516987.PDF [Page: 6 of 39]

104-06263/

Abstract

A Monte Carlo study of the propagation lives of through-thickness cracks in ship's deck plate subjected to mode-I deformation and their sensitivity to various parameters was carried out. The simulation model is based on the Paris law, and takes into account those uncertainties that affect demand (stress-intensity factor) and capability (fracture toughness). The effects of load sequence on the crack-growth rate were ignored, and the random loading on the hull girder was approximated by equivalent constant-amplitude loading. The results are presented in plots of the statistical estimate of the propagation life versus the initial crack length. The propagation life was found to be relatively insensitive to the initial crack length when the latter was greater than about 10 cm but sensitive to the applied stress range.

Résumé

On a effectué une étude Monte Carlo sur la vitesse de propagation des fissures traversantes sur les tôles de pont de navires soumis aux déformations de mode I et sur leur sensibilité à divers paramètres. Le modèle de simulation est basé sur la loi de Paris et tient compte des incertitudes qui influent sur la demande (coefficients de contrainte et d'intensité) et les capacités (ténacité). Les effets de la séquence de charge sur le coefficient de croissance des fissures a été ignoré et les charges aléatoires sur la poutre-coque ont été évaluées approximativement par équivalence avec les charges à amplitude constante. Les résultats sont présentés sous forme de tracés des estimations statistiques de la vitesse de propagation par rapport à la longueur initiale de la fissure. On a remarqué que la vitesse de propagation était relativement insensible à la longueur initiale de la fissure lorsque celle-ci était supérieure à 10 cm, mais qu'elle était sensible à la plage des contraintes exercées.

This page is left blank

This page is left blank

P516987.PDF [Page: 8 of 39]

Executive summary

Fatigue cracks are potentially the main danger for the structural safety of ships. As such, an adequate fatigue-control plan is an important factor in the Improved Ship Structural Maintenance and Management program (ISSMM) currently under development by the Department of National Defence. The goal of ISSMM is to develop a cost-effective methodology for ensuring the structural safety of the Canadian Navy's ships throughout their service lives. As part of the investigation of fatigue performance of ship structures, Monte Carlo simulations of the crack propagation life were conducted in this study. The objective is to obtain statistical estimates of the length of time a crack discovered on the deck plate near amidships would take to grow to a critical length. The uncertainties about parametric values of the governing equation (Paris law) as well as applied loads and structural capability were taken into account.

The plots of the initial crack length versus the propagation life at 0.95 probability (that is, the probability of the crack's growing to the critical length in greater than the indicated number of stress cycles is 0.95) were obtained. The propagation life was seen to be relatively insensitive to the initial crack length when the latter was greater than about 10 cm. Qualitatively, the results underscored the importance of a well-known fact that the propagation life is sensitive to the stress range (i.e., the difference between maximum and minimum stress levels).

Apart from the appropriateness of the theoretical model, the reliability of any statistical study depends on that of the input data. Thus, given the inherent scatter of the fatigue test samples, it may be worthwhile to carry out further tests and enlarge the existing database for the materials used in the structures of Canadian Forces ships. Equally important in reducing the margin of error are efforts towards obtaining improved knowledge of service stresses. Monte Carlo studies of the following problems appear worthwhile, considering their probabilistic nature: long-term fatigue damage, propagation lives of cracks initiating at a corner and welding, the mutually enhancing effects of corrosion and fatigue, and the effects of load sequence (or load interaction) and load variability.

Ando S. 2001. Monte Carlo Simulation of Crack Growth on Ship Hull Plating. DREA TM 2001-166 Defence Research Establishment Atlantic.

516987.PDF [Page: 9 of 39]

Sommaire

Les fissures de fatigue sont susceptibles de constituer le principal danger pour la sécurité structurale des navires. C'est pourquoi le programme de Gestion améliorée de la maintenance des structures de navires (GAMSN), actuellement mis au point par le ministère de la Défense nationale, laisse une place importante à un plan de contrôle de la fatigue. Le but du GAMSN est d'élaborer une méthode rentable d'assurer la sécurité structurale des navires de la marine canadienne pendant toute leur durée de vie. Dans le but d'examiner la résistance à la fatigue des structures de navires, des simulations Monte Carlo de la vitesse de propagation des fissures ont été effectuées lors de cette étude. L'objectif est d'obtenir des estimations statistiques du temps que prend une fissure découverte sur une tôle de pont à mi-navire pour atteindre une longueur critique. On a pris en compte les incertitudes relatives aux valeurs paramétriques de l'équation principale (loi de Paris) de même que les charges appliquées et la capacité structurale.

On a obtenu des tracés de la longueur initiale de la fissure par rapport à la vitesse de propagation à une probabilité de 0,95 (c'est-à-dire, la probabilité que la propagation de la fissure jusqu'à la longueur critique dépasse le nombre de cycles de contrainte indiqués est de 0,95). On a remarqué que la vitesse de propagation était relativement insensible à la longueur initiale de la fissure lorsque celle-ci était supérieure à 10 cm. Du point de vue qualitatif, les résultats révèlent l'importance d'un fait bien connu, à savoir que la vitesse de propagation est sensible à la plage des contraintes (c.-à-d. la différence entre les niveaux de contrainte maximum et minimum).En plus de la justesse du modèle théorique, la fiabilité de toute étude statistique dépend de la fiabilité des données initiales. C'est pourquoi, étant donné la dispersion des éprouvettes d'essais de résistance à la fatigue, il pourrait être utile d'effectuer d'autres essais et d'élargir la base de données existante aux matériaux utilisés pour les structures des navires des Forces canadiennes. Pour réduire la marge d'erreur il serait aussi important d'approfondir les connaissances sur les contraintes en service. Des études Monte Carlo sur les problèmes suivants pourraient être intéressantes, étant donné leur degré de probabilité : dommages à long terme causés par la fatigue, vitesse de propagation des fissures débutant dans une soudure d'angle, effets combinés de la corrosion et de la fatigue et effets de la séquences de charge (ou interaction des charges) et de la variabilité des charges.

Ando S. 2001. Monte Carlo Simulation of Crack Growth on Ship Hull Plating. DREA TM 2001-166 Defence Research Establishment Atlantic.

P516987.PDF [Page: 10 of 39]

Table of contents

Abstract i

Executive summary iii

Sommaire iv

Table of contents v

List of figures vi

List of tables vi

Acknowledgements vii

Introduction 1

Theoretical model for prediction of fatigue life 2

Results and discussion 9

Concluding remarks 12

References 17

Annexes 18

List of symbols/abbreviations/acronyms/initialisms 23

516987.PDF [Page: 11 of 39]

List of figures

Figure 1. Modes of crack surface displacement 14

Figure 2. Crack of length $2a$ in plate of width W under random loading 14

Figure 3. Fifth percentile of propagation life vs. initial crack length for two values of σ_{max} 15

Figure 4. Fifth percentile of propagation life vs. initial crack length for two values of K_c 16

Figure B1. Lognormal distribution:($(\mu,\sigma) = (7.23\text{x}10^{-11}, 1.0\text{x}10^{-11})$ 19

Figure B2. Truncated Normal distribution: $(\mu,\sigma,x_{min},x_{max}) = (3.0, 0.1, 2.85, 3.05)$ 20

Figure B3. Rayleigh distribution, $b = 60, 100$ 20

Figure C1. Relative frequency of N_{005} from 1000 samples for a crack of initial length of 10 cm 21

Figure D1. Propagation life versus initial crack length 22

List of tables

Table 1. Average value of yield strength and ultimate strength for ASTM A517 Grade F steel and CSA G40.21 Grade 350WT steels at strain rate 0.0001 s^{-1} 13

P516987.PDF [Page: 12 of 39]

Acknowledgements

This study was initiated at the request of DMSS2-2 (Lt Cdr A.R. Graham). I'd like to thank John Porter of DREA Dockyard Lab for going over the metallurgical aspect of this report and making a number of useful suggestions for clarity and perspicuity.

P516987 PDF [Page: 14 of 39]

Introduction

The Department of National Defence (DND) is developing a cost-effective methodology for maintaining the structural safety of the Canadian Navy ships throughout their service lives under the Improved Ship Structural Maintenance and Management (ISSMM) program. For ISSMM Project Manager (PM), knowledge of the extents of the risk that would arise from errors and uncertainties attending various components of the ISSMM is indispensable in his/her effort to ensure the reliability of ISSMM software and to guide future improvements. Errors in the following components of ISSMM are expected to be major sources of risk.

- wave statistics and application methodologies,
- prediction methods for sea loads,
- prediction methods for structural responses,
- strength and fatigue data of materials, fracture toughness properties,
- prediction methods for fatigue life (initiation and growth), and
- residential strains.

Obviously, extensive work is required to examine all the items, and the plan is to approach the task in an incremental fashion. This report describes a "first-cut" investigation of crack growth carried out as part of the study of fatigue performance of ship structures. The objective is to incorporate the uncertainties about the loads and capability in the predicted fatigue life by the Monte Carlo method. This approach does not require as comprehensive (and hard-to-obtain) data or as laborious execution as does the ISSMM software package, which is designed to be general enough to handle the full spectrum of structural analysis. The Monte Carlo method thus offers an economical alternative to ISSMM. This is DMSS2-2's main motivation for suggesting this study.

Metal fatigue is a phenomenon in which a crack initiates and grows in a structural component exposed to repeated loading above a certain threshold level. (The latter is below the tensile strength of the material.) Ship hulls and offshore platforms are exposed to cyclic loading by ocean waves, and are therefore vulnerable to fatigue failure. Fatigue is a complicated metallurgical phenomenon. It depends on such factors as size-effects, stress concentration, order of loading of varying stresses, stress range, overall design, operating temperature of a structure, as well as material and material homogeneity. For long cracks, bulk properties and bulk loading conditions also influence their growth. A fatigue crack almost invariably begins at a root of a geometric discontinuity where a stress concentration occurs. Once started, it becomes a travelling stress concentration and tends to enhance the growth. If unattended, a fatigue crack could lead to a catastrophic failure of a structure, including the loss of a ship. The case of the Liberty ships is a well-known example [1]. There are other modes of structural failure that must be taken into account to establish the projected safe life for Canadian Forces ships. They include yielding, plastic instability, buckling, corrosion, stress-corrosion cracking, creep, and brittle fracture. But fatigue is perhaps the most important failure mechanism, for it is estimated that fatigue causes between 85 to 90 percent of all failures. Thus, fatigue warrants special consideration.

Fatigue life comprises two components:

$$\text{Fatigue life} = (\text{Initiation life}) + (\text{Propagation life}).$$

P516987.PDF [Page: 15 of 39]

The initiation life refers to the onset and early growth of a small crack. The propagation life encompasses the duration for the crack to grow to a critical length, that is, a size that poses a threat to structural integrity. Usually, the initiation life covers the time span for formation of a crack about 2 to 3 mm long, which is readily detected with nondestructive inspection equipment. But there are no definite transition criteria from initiation to propagation; they are decided on a case-by-case basis, taking into account the size of the component being analyzed. For large structures like ship hulls, the propagation life dominates fatigue life. Thus, the objective here is to address the following questions using Monte Carlo simulation.

- How long will a crack discovered on a deck plate of the HALIFAX-class frigate take to grow to the critical length under normal service conditions?
- To which factor(s) is the fatigue life sensitive?

This study is intended as a "first-cut" investigation of crack-growth life and of sensitivity of the crack growth to various parameters. As such, it is not the intent of this study to strive for rigor, and the dearth of relevant data is to be augmented by educated guesswork wherever necessary. The result is to help refine the ISSMM program by shedding some light on which of its components warrant further investigation.

Fatigue often involves complex stress states (i.e. the three principal stresses are non-proportional or whose directions change during a loading cycle) at geometric discontinuities such as notches or joint connections. Fatigue under these conditions, termed multiaxial fatigue, is an important design consideration for reliable operation and optimization of many engineering components. But the research on multiaxial fatigue is still in large part in a developmental stage, although it continues to be a topic of concentrated research. In this study, therefore, the well-established uniaxial loading conditions are assumed.

There are generally three modes of loading and crack-surface displacements (Fig. 1): the opening mode (mode I), the sliding or shear mode (mode II), and the tearing mode (mode III). This study focuses on mode I, since it is the mode most likely to be involved in failure of hull plating. The assumptions in this study are that the hull plate is under uniform tension and that a crack penetrates through the thickness of the plate.

There are two fatigue models commonly used. One is based on fracture mechanics and the other on the Palmgren-Miner rule. This study uses the former. By characterizing sub-critical crack growth using linear elastic fracture mechanics (LEFM), it is possible to predict crack growth rates under cyclic loading, and hence the number of cycles necessary for a crack to extend from some known initial size to a maximum permissible size, in other words, the propagation life. On the other hand, the Palmgren-Miner rule is based on fatigue-test data (S-N curves) and the assumption of linear accumulation of fatigue damage. The model is therefore predicts the total fatigue life, including the initiation life.

Theoretical model for crack growth

The underlying assumptions for LEFM are [1, 2]:

- that cracks are in regions where the nominal stresses are elastic; and

P516987 PDF [Page: 16 of 39]

- that cracks are large enough compared with microstructural features of the metal so that the concepts of continuum mechanics are applicable.

The advantage of LEFM is that the magnitude of local stress σ around the crack-tip can be described by a single parameter, the stress-intensity factor K, which has units of $\text{MPa}\sqrt{\text{m}}$ or $\text{ksi}\sqrt{\text{in.}}$ (1 $\text{ksi}\sqrt{\text{in.}}$ = 1.0988 $\text{MPa}\sqrt{\text{m}}$ = 34.75 $\text{MPa}\sqrt{\text{mm}}$). For mode-I deformation, K has the general form:

$$K = Q\sigma\sqrt{a}\,, \tag{1}$$

where a denotes the crack half-length (Fig. 2) and Q is a dimensionless parameter used to account for the type of crack loading, and the ratio of crack size to specimen or component dimensions. For the specific case under consideration—a through-thickness crack of the length $2a$, which is located away from edges of the plate subject to uniaxial tension (Fig. 2)—the stress-intensity factor is expressed as

$$K = \sigma\sqrt{\pi a}\,. \tag{2}$$

The design stress level that can be safely used for an existing crack can also be determined from (1) or (2) in case of mode-I displacements.

If a relevant S-N curve for material is available, fatigue failure in service can be avoided by either keeping the cyclic stresses below the fatigue strength, or ensuring that the number of cycles in service is less than the fatigue life at the same stress level. But the loads on a ship induced by waves and by the wave-induced motions of the ship vary randomly, and the actual service load history is usually unknown (although some ships are now fitted with strain gauges to record it). So the requisite data for prediction of fatigue life are not only scarce but also subject to a great deal of uncertainties.

The statistical simulation by the Monte Carlo method provides a rational means to obtain insights in complex phenomena beset with uncertainties. Crack propagation is such a phenomenon. The Monte Carlo method is also a useful alternative to an elaborate sensitivity analysis using the ISSMM program. Designed to handle the full spectrum of structural analysis, the latter requires comprehensive (and hard-to-obtain) data like derivatives of structural response quantities (e.g. stress, displacement, buckling load, natural frequency) with respect to the design variables (usually scantlings of hull structural members). It also entails laborious execution of finite-element programs. As an economical alternative, the Monte Carlo method is used in this study to simulate the crack propagation life using a simple model based on the Paris law. The results highlight some significant features about the crack propagation life, including its relative insensitivity of propagation life to the initial length when the latter is greater than about 10 cm or so and its sensitivity to the applied stress range.

The fatigue life is taken as the number of cycles N required for a crack to propagate from an initial size to a critical size that can initiate unstable growth. The simulation of fatigue life in this study consists of three steps:

1. measurement of the initial crack size $2a_i$,
2. determination of the maximum permissible crack size $2a_c$, and
3. evaluation of the period of crack growth between a_i and a_c.

P516987.PDF [Page: 17 of 39]

where a_i and a_c denote the initial and critical half-lengths of a crack for given load cycle(s). In this study, the Monte Carlo method is used to simulate steps 2 and 3 for through-thickness cracks of such initial sizes as detectable by non-destructive inspection methods. In its simplest model, a ship can be regarded as a girder made up of the upper deck, bottom shell, side plating, and longitudinal bulkheads. Here, the ship hull girder is regarded to be one large test specimen, which is subjected to alternating bending by waves. For simplicity, the effects of the operating temperature and of corrosion are neglected.

The Monte Carlo simulations (see, e.g. [3]) are essentially a numerical method for estimating the unknown value of some parameter of certain distribution. Let $f(x)$ and $F(x)$ denote the probability density function (pdf) and the cumulative distribution function (cdf) of the random variable X. (Although the term "random" means "unpredictable" in the sense that the exact value cannot be predicted, some characteristic value(s) like the average value and some sense of dispersion of the aggregate properties of the population are known or assumed to be known in terms of a distribution function.) Then the essence of the Monte Carlo simulation of X can be expressed by

$$x = F^{-1}(p), \tag{3}$$

where p is a number between 0 and 1 such that

$$p = P[X \le x] = \int_{-\infty}^{x} f(y)dy. \tag{4}$$

The steps involved in the Monte Carlo simulations are as follows:

1. generate realizations using given probability laws (pdf's),
2. solve the deterministic problem for large number of realizations of the input, and
3. analyze the results statistically.

The largest bending stress induced by waves in the hull girder occurs near amidships. Customarily, the midship scantlings are maintained throughout the midship four-tenths length of the hull. In this study, the primary load-carrying members of ship structures are assumed to be the plate members within the midship four-tenths length of the hull. The stiffeners, although they can also be primary load-carrying members, are ignored because they are not connected to each other so that unlike the shell plating, failure of one stiffener should not lead to failure of adjacent ones. The focus in this study is the propagation life of a crack that is located near amidships and that runs transversely. (In the case of the HALIFAX class frigates, the center of the funnel is situated almost at amidships.)

Here, the tensile load is used as the representative random variable to arrive at a simulation model. Thus the predominant loading mode in the structural failure of hull girder is assumed to be the deck plate subjected to mode-I deformation (tensile stresses normal to the plane of the crack). Because of the through-thickness uniformity of strain on hull plate, this model, in spite of its simplicity, is a realistic one even though a ship hull under service conditions is subjected to combined bending and torsional moments under the influence of corrosion and thermal effects.

P516987.PDF [Page: 18 of 39]

For mode-I crack, fatigue-crack-growth rate per cycle of loading is conveniently represented by the Paris law:

$$\frac{da}{dN} = C(\Delta K)^m, \tag{5}$$

where da/dN is the cyclic crack growth rate, ΔK is the stress-intensity factor range, and C and m are material parameters experimentally determined. The stress-intensity factor range ΔK is defined as

$$\Delta K = K_{max} - K_{min}, \tag{6}$$

where K_{max} and K_{min} are the maximum and minimum values of the stress-intensity factor K. Let σ_{max} and σ_{min} denote respectively the maximum and minimum values of far-field stresses during a fatigue cycle. For mode-I crack,

$$\begin{Bmatrix} K_{max} \\ K_{min} \end{Bmatrix} = \begin{Bmatrix} \sigma_{max} \\ \sigma_{min} \end{Bmatrix} \sqrt{\pi a}, \tag{7}$$

rom (2), and so

$$\Delta K = \Delta\sigma\sqrt{\pi a}, \tag{8}$$

where $\Delta\sigma$ denotes the stress range:

$$\Delta\sigma = \sigma_{max} - \sigma_{min}. \tag{9}$$

One of the shortcomings of (5) is that the growth rate prediction is limited to the linear part of the $\log da/dN$ versus $\log \Delta K$ curve. Another is that the growth rate does not depend on the mean stress, which is contrary to experimental findings. Nevertheless, (5) has been established experimentally to be valid for a wide range of growth rates, between 10^{-6} and 10^{-3} mm/cycle, and is commonly used by design engineers.

The applied stress ratio R, where

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{K_{min}}{K_{max}}, \tag{10}$$

can have a significant effect on both the crack-growth rate and the fatigue threshold. There are a number of fatigue-crack growth-rate models that modify the expression (5) explicitly to take the R-ratio into account. Notable among them are Forman's equation [4] and Walker's equation [5]: see Annex A. Forman's equation is valid only for $R > 0$. The hull girder usually experiences alternating sagging and hogging stresses as a ship traverses in waves. Thus R may be positive or negative, depending on the magnitude of the residential stresses. On the other hand, Walker's equation is not appropriate for this study, since the exponents m and n in the equation can only be determined by fitting the experimental data and no such data are currently available.

P516987.PDF [Page: 19 of 39]

For ships, the crack growth in the low ΔK region is important, because this is the region that comprises most of their fatigue life times. But ΔK must be greater than some threshold value ΔK_{th} for a crack to grow. For the ASTM A517 Grade F steel and CSA G40.21 Grade 350WT steel, which are respectively used for No. 1 deck and for most of the rest of the hull structural steels of the HALIFAX-class frigate, reference [6] gives a threshold value of $\Delta K_{th} = 63\ \text{MPa}\sqrt{\text{mm}}$ in both air and sea environments for all loading ratios when residual stress levels are unknown.

For deck plating of a ship, the load spectra usually consist of predominantly tension-compression cycles (R < 0). Under these circumstances, the neglect of the load-interaction effects on crack growth (e.g. retardation and acceleration) tends to result in a shorter predicted fatigue life under constant-amplitude cyclic load fluctuation than that under variable-amplitude load spectra having the same value of σ_{rms} [7]. Although negative loads in a constant amplitude test cause practically no interaction effects, they are detrimental in an indirect way in that they partly annihilate the residual stresses built up by positive overloads, thus reducing the latter's beneficial effect. In this study, the wave-induced loading on the hull girder is assumed to be cyclic, and $R = -1$.

But it is known that the growth rates when $R < 0$ and when $R = 0$ are not significantly different. Furthermore, the increases in growth rate relative to $R = 0$ as a result of negative R are less than a factor of two. Thus, crack-growth life predictions are typically made only for the tensile portion of loading cycles. Given all the uncertainties involved in making fatigue life and crack growth predictions, neglecting the effects of compressive load may often be justified, especially in preliminary analysis.

Other factors influencing the crack growth rate include the environmental conditions, welding and temperature. For example, for the A517 and 350WT steels, the fatigue crack growth rates in sea water were found to be two to three times greater those in air at $\Delta K =$ 1000 $\text{MPa}\sqrt{\text{mm}}$, and crack growth rates for weld metals were up to twice those for the A517and 350WT base metals [6]. Temperature affects the yield strength and the ultimate strength of steel. Tensile test results with theA517 and 350WT steels at 0°C and -40°C are shown in Table 1 [6]. In the context of fatigue life, however, perhaps the more significant temperature effect will be on the critical crack length.

Based on the results of fatigue tests with specimens, reference [6] suggests the following form of the Paris law for ferritic-pearlitic structural steels for the A517 and 350WT steels exposed to seawater:

$$\frac{da}{dN} = 2.3\times10^{-12}(\Delta K)^{3.0}, \tag{11}$$

for all stress ratios R, where da/dN is in mm/cycle and ΔK is in $\text{MPa}\sqrt{\text{mm}}$. Note that reference [2] gives the following expression for a number of martensitic steels, including the ASTM A517 Grade F steel:

$$\frac{da}{dN} = 0.66\times10^{-8}(\Delta K)^{2.25}, \tag{12}$$

P516987.PDF [Page: 20 of 39]

where da/dN is in in./cycle and ΔK in ksi$\sqrt{\text{in.}}$ In this study, (11) is used as the basis for the Monte Carlo simulation. Since the deck is not exposed to seawater constantly, the proportional constant 2.3×10^{-12} in (11) may be reduced by a factor of two or three. The results are, therefore, expected to be conservative.

Let K_{I_c} denote the mode-I fracture toughness, or the critical value of K at which unstable crack growth occurs for a given material of a particular thickness and at a specific temperature and slow loading rate. As (1) indicates, the fracture toughness K_{I_c} is a material property, an inherent attribute of the plate used, and depends on such things as the manufacturing process, plate thickness, welding process, temperature, and strain rate. The critical crack half-length a_c that can be tolerated at a given maximum stress σ_{max} can be calculated from (1). In the case of a through-thickness crack in the plate in tension, (2) gives

$$a_c = \frac{1}{\pi}\left(\frac{K_{I_c}}{\sigma_{max}}\right)^2. \tag{13}$$

The discussions so far are concerned with constant-amplitude cyclic loads. But the loads exerted by ocean waves on the ship hull are time-varying. Under such variable-amplitude loading, the magnitude of ΔK changes for each cycle. Nevertheless, for some types of variable-amplitude loads, notably narrow-band random loading, it turns out that the crack growth is a fairly regular process and that crack growth rates can be predicted reasonably well from constant-amplitude data by (see, e.g. [8,9])

$$\frac{da}{dN} = C(\Delta K_{rms})^m, \tag{14}$$

where ΔK_{rms} is the root-mean-square of the individual load cycles in a spectrum such that

$$\Delta K_{rms} = \sqrt{\frac{\sum_{j=1}^{n}\Delta K_j^2}{n}}, \tag{15}$$

and n is the number of cycles in an observation period. From (8), ΔK for the jth cycle of a through-thickness crack in mode I is given by

$$\Delta K_j = \Delta\sigma_j\sqrt{\pi a_j}. \tag{16}$$

Equation (14) is based on a statistical average (the rms value). This method may not be suitable for situations other than load sequences that can be described as a stationary narrow-band Gaussian process with a unimodal distribution. For crack growth under random loading, stress spectra are commonly represented by a continuous, unimodal distribution, usually by a Rayleigh distribution. Since well-developed ocean waves have narrow-band spectra, and since the wave-ship system is very closely linear, it is reasonable to assume that loads induced

P516987.PDF [Page: 21 of 39]

by waves on the ship hull also possess narrow-band spectra, and that crack growth is a regular process. In this study, therefore, its rate is predicted by using the appropriate value of the effective stress range $\Delta\sigma_{eff}$, which is assumed to be constant for each stress cycle [10] and equal to the rms value.

Since the deck plating is subjected to alternating hogging and sagging moments, both individual load cycles and their interaction should be accounted for. The load-interaction effects include tensile-overload retardation, compressive-load acceleration, and the reduction of overload retardation effect caused by compressive load immediately following the tensile overload [7]. But it is difficult and complicated to define and count stress cycles for the ship hull girder. For one thing, the wave-induced stresses are essentially random, and for another, they include occasional subcycles of relatively small amplitude caused by impulsive loads like bottom slamming and bow-flare impact, which are interspersed in the underlying relatively large-amplitude, slow-varying wave-induced load cycle. But such subcycles do very little fatigue damage because not all of them give rise to such stress magnitudes as exceed the threshold stress level and add to extra cycles. In this study, therefore, a non-interaction model is used, so that each cycle is assumed to do a given amount of damage independent of any prior history. Another basic premise in this study is that a fatigue life based on a constant-amplitude history ($\Delta\sigma = \Delta\sigma_{eff}$ = const.) provides a reasonable estimate for the variable-amplitude loading so that the increment in crack size during one load cycle may be computed from an equation for constant-amplitude loading.

One method to simulate the crack growth per cycle for the stochastic crack propagation under variable-amplitude loading (assuming a noninteraction model) is by

$$a_j = a_i + \sum_{k=1}^{j} \Delta a_k \,. \tag{17}$$

The crack growth Δa_k in the kth stress cycle is found by

$$\Delta a_k = C(a_k)(\Delta\sigma_k)^m \left(\sqrt{\pi a_k}\right)^m, \tag{18}$$

where an appropriate value of $\Delta\sigma_k$ is inserted for each stress cycle. Equation (18) is solved for $j = 0, 1, 2, \ldots$, until the accumulated crack length reaches the critical length $a_c = a_f$ for some j. But the focus in this study is the propagation life of a crack of given initial length, so it is just as well to make use of the exact integral of (5). The effect of variable-amplitude loading is taken into account by using the effective stress intensity factor range, which, for a crack in an infinite panel in uniaxial tension (Fig. 2), is defined as follows:

$$\Delta K_{eff} = \sqrt{\pi}\Delta\sigma_{eff}\sqrt{a} \,. \tag{19}$$

From (5), the number of load cycles for the crack size to increase from a_i to a_f is found by

$$\int_0^N dv = \int_{a_i}^{a_c} \frac{da}{C(\Delta K_{eff})^m} \,. \tag{20}$$

P516987.PDF [Page: 22 of 39]

It is assumed here that $\Delta\sigma_{eff}$ = constant, so (20) can be written as

$$N = \frac{1}{\left(\sqrt{\pi}\Delta\sigma_{eff}\right)^m}\int_{a_i}^{a_c}\frac{da}{C(a)a^{m/2}}. \tag{21}$$

When C varies randomly as a function of crack length a, (21) is a stochastic integral and has to be integrated numerically. Here C is assumed to be independent of a; thus (21) can be integrated analytically to give the required number of load cycles exactly. It will be sufficient to consider the case for $m > 2$, since the experimental values of m for steel fall between 2.2 and 3.3, approximately (see e.g. [2, 3]). From the expression for $m > 2$ in (21), the crack growth life for this case is given by

$$N = \frac{1}{C\left(\sqrt{\pi}\Delta\sigma_{eff}\right)^m}\frac{a_c^{1-\frac{m}{2}} - a_i^{1-\frac{m}{2}}}{1-\frac{m}{2}}. \tag{22}$$

Substituting (13) for a_c in (22) yields

$$N = N(C, K_c, m, \Delta\sigma_{eff}, \sigma_{max}; a_i) = \frac{1}{C\left(\sqrt{\pi}\Delta\sigma_{eff}\right)^m}\frac{\left[\frac{1}{\pi}\left(\frac{K_c}{\sigma_{max}}\right)^2\right]^{1-\frac{m}{2}} - a_i^{1-\frac{m}{2}}}{1-\frac{m}{2}}. \tag{23}$$

In this study, (23) was used as the basis of simulation of the propagation life N. The approach was to consider the propagation life corresponding to a given initial crack length as a random process and obtain its statistical properties. This was done by substituting for the variables and parameters, C, K_{I_c}, m, $\Delta\sigma_{eff}$, and σ_{max}, in (23) a corresponding set of random values having the prescribed probability distributions shown in the next section.

Results and discussion

The following distributions are assumed for the parameters and variables in the simulation of the propagation life as expressed in (23). The symbol $\in$ indicates that the parameter on its left takes on the values listed inside the brackets, one at a time. For the probability distributions, see Annex B for their mathematical expressions, plots, and explanations of the parameters inside parentheses. The units are m for length and MN for force.

P516987.PDF [Page: 23 of 39]

$$
\begin{aligned}
&K_c \in \{100, 150\},\\
&C = \text{Lognormal distributed with } (\mu,\sigma) = \left(7.23\times10^{-11}, 1.0\times10^{-11}\right),\\
&m = \text{Truncated-Normal distributed with } (\mu,\sigma,x_{\min},x_{\max}) = (3.0, 0.1, 2.85, 3.05),\\
&\sigma_{\max} = \text{Rayleigh distributed with mode } b \in \{50,70\},\\
&R = -1, \text{ and}\\
&\Delta\sigma_{\text{eff}} = \sqrt{2}\sigma_{\max}.
\end{aligned}
\tag{24}
$$

Here C, m, and $\sigma_{\max}$ are assumed to be independently distributed. Their values for a given combination of a_i and K_{I_c} were sampled randomly from the mathematical distributions specified in (24). The Latin hypercube sampling was used in preference to the traditional Monte Carlo technique. The former has been shown [11] to be more efficient than the latter, which uses pseudo-random numbers to sample from a probability distribution. The mode—the most frequently occurring value—of the distribution for the constant C is 2.3×10^{-12} when da/dN is mm/cycle and ΔK in MPa$\sqrt{\text{mm}}$; see (11). The last line of (24) follows from the assumptions of the constant-amplitude load cycles and $R = -1$, and from the fact that $\Delta\sigma_{\text{eff}}$ is set to equal to the rms value in this study. Note that the stress level is time-varying, so the value of $\sigma_{\max}$ and $\Delta\sigma_{\text{eff}}$ depend on the length of the time interval over which measurements are to be taken. The two values of the fracture toughness K_{I_c} and of the Rayleigh parameter b used for simulation were so chosen that they bracket the extreme values of the respective parameters for the HALIFAX-class ships.

Figures 3 and 4 summarize the main results of this study. They show the 5$^{\text{th}}$ percentile $N_{0.05}$ of propagation life N, versus the initial crack length $2a_i$. The 5$^{\text{th}}$ percentile means that the probability that a newly discovered crack of half-length a_i grows to the critical half-length a_c (13) in less than $N_{0.05}$ cycles is 0.05. Here, $N_{0.05}$ was obtained from the set of 10,000 simulated load histories. A histogram such as shown in Annex C approximates the probability distribution of N for a particular value of a_i. (A sample size of 10,000 was considered sufficient to yield enough data, since the differences among the predicted values of N's as a results of 5,000, 10,000, and 20,000 samples were found to be less than one percent of the predicted N.) Each line in these figures connects $N_{0.05}$ calculated at the following 18 values of initial half-crack length a_i (in m):

$$
\begin{aligned}
a_i \in \{&0.0025, 0.005, 0.01, 0.012, 0.015, 0.02, 0.03, 0.04,\\
&0.05, 0.06, 0.07, 0.08, 0.09, 0.10, 0.12, 0.15, 0.20, 0.25\}
\end{aligned}
\tag{25}
$$

The dotted lines in the plots show the percent decreases between the two values of N relative to the larger value represented by the broken lines.

Plots of $N_{0.05}$ versus the initial crack length $2a_i$ when $\sigma_{\max}$ is Rayleigh distributed with mode b = 50 MPa and 70 MPa are shown in Figs. 3a and 3b, respectively. In these figures, the solid line indicates the $N_{0.05}$ for $K_c = 100$ MPa$\sqrt{\text{m}}$ and the broken line for

P516987.PDF [Page: 24 of 39]

$K_c = 150$ MPa$\sqrt{\text{m}}$. When $b = 50$ MPa (Fig. 3a), $N_{0.05}$ ranges from 36,999 at $2a_i = 0.5$ cm to 1,156 at $2a_i = 50$ cm for $K_c = 100$ MPa$\sqrt{\text{m}}$, and from 38,448 at $2a_i = 0.5$ cm to 2,635 at $2a_i = 50$ cm for $K_c = 150$ MPa$\sqrt{\text{m}}$. When $b = 70$ MPa (Fig. 3b), $N_{0.05}$ ranges from 12,533 at $2a_i = 0.5$ cm to 281 at $2a_i = 50$ cm for $K_c = 100$ MPa$\sqrt{\text{m}}$, and from 13,145 at $2a_i = 0.5$ cm to 312 at $2a_i = 50$ cm for $K_c = 150$ MPa$\sqrt{\text{m}}$. The dotted line in Fig. 3a shows that when $b = 50$ MPa, the difference between the two values of $N_{0.05}$ corresponding to $K_c = 100$ MPa$\sqrt{\text{m}}$ and 150 MPa$\sqrt{\text{m}}$ ranges from 4 to 56 percent relative to the values corresponding to $K_c = 150$ MPa$\sqrt{\text{m}}$ over the range of the initial crack length up to 50 cm. When $b = 50$ MPa (Fig. 3b), the difference is 5 to 52 percent, with maximum difference occurring over 16 cm $\leq 2a_i \leq$ 20 cm. When subjected to the same load level, the values of $N_{0.05}$ for the two different values of K_c differ very little for small initial crack length ($2a_i$), say, up to about 3 cm.

In Fig. 4, plots of $N_{0.05}$ versus the initial crack length $2a_i$ for two values of the fracture toughness $K_c = 100$ MPa$\sqrt{\text{m}}$ (solid line) and 150 MPa$\sqrt{\text{m}}$ (broken line) are shown. Figure 4a shows the case where σ_{max} is Rayleigh distributed with mode $b = 50$ MPa and Fig. 4b, the case where σ_{max} is Rayleigh distributed with mode $b = 70$ MPa. In these figures, the difference between the values of $N_{0.05}$ represented by the broken line and solid lines are seen to amount to 66 to 88 percent relative to the values represented by the broken line. This underscores the well-known fact that the stress range (the difference between maximum and minimum stress levels) is a very important parameter in controlling the fatigue life of structural components.

For most ships, the wave-induced loads are the main source of cyclic stress in their deck plates around amidships, and the number of the wave-bending cycles in a ship's life (20 years or thereabouts) is conventionally assumed to be of the order of 10^8 cycles. But as mentioned previously, not all wave encounters give rise to large enough ΔK above its threshold value, let alone σ_{max} of 50 to 70 MPa. (Probabilities of these load parameters exceeding certain prescribed values need to be addressed in the future study.)

Other things being equal, a large increase in initial crack length results in a relatively small amount of decrease in the propagation life for the initial crack length greater than 10 cm or so, as the slope of the curves in Figs. 3 and 4 indicates. For example, a 500-percent increase in the initial crack length from 10 cm to 50 cm results in about 17 to 34 percent decreases in the propagation life in these figures. Note that Figs. 3 and 4 represent one set of 10,000 realizations per initial crack length. If another set of 10,000 realizations were generated, a histogram such as shown in Annex C would be similar in shape but different in detail; hence the curves in Figs. 3 and 4 would shift somewhat. Annex D provides some idea of the magnitude of the shift pertaining to these figures.

Figures 3 and 4 are based on the rms stress-factor range, which, strictly speaking, is valid for random loading having a narrow-band spectrum that does not contain occasional high tensile loads. But the deck plating near amidships of a ship is sometimes subjected to high, impulsive tensile loads when the ship hogs, for example, when the bow is lifted out of water as a result of large pitching motion in severe waves. Breaking of green sea during the

P516987.PDF [Page: 25 of 39]

subsequent downward motion of the bow may further intensify the tensile loads in the deck plating amidships. Because extreme loads are most likely to be caused by these localized impulsive forces, their characteristics need to be better understood to improve the reliability of hull structural analysis. In a future study, therefore, the extent of the influence of these occurrences needs to be examined.

Concluding remarks

In this study, propagation life of a through-thickness crack in the deck plate of a ship hull was simulated using the Monte Carlo method. In the simulation model, the ship hull girder was assumed to be undergoing alternating bending by waves.

The results provided some insight in the fatigue life of the HALIFAX-class frigate. For example, the propagation life of a crack was found to be not very sensitive to the length of the initial crack when the initial crack length was greater than 10 cm or so. On the other hand, the results underscored the well-known sensitivity of the crack propagation life to the applied stress range.

One limitation of this study relates to the uncertainties about the data on fatigue properties of the hull structural steels and about service loads. No reliability-based structural model is possible without reliable data about the demand and capacity. So more should be known about the material properties and service stresses. The fatigue tests of the steels summarized in [6] are part of the effort to address the capacity aspect of the problem. For the latter, onboard measurements by strain-gauge and efforts to develop a reliable probabilistic model of service loads are both essential.

Another major limitation of this study lies in the fact that the effects of corrosion, operating temperature, and load interaction were neglected. In the harsh deleterious environments in which ships operate, corrosion is practically unavoidable and the conjoint (i.e., mutually assisting) actions of fatigue and corrosion (stress-corrosion cracking) have serious effects on fatigue life of the structure and need to be addressed in future studies.

But qualitatively at least, the present results provide an insight into fatigue phenomena, which helps us to focus on specific areas for future extensions and refinement of fatigue analysis. For example, a Monte Carlo simulation of effects of hydrodynamic impulsive loads on hull structural components will be worthwhile, since ships in general are essentially single-load path structures rather than multiple-load path structures like most bridges, and as such, their structural integrity tends to be more vulnerable to weakness in components. As well, a Monte Carlo study on the propagation lives of cracks initiating at a corner and/or weldment is of immediate interest, for the fatigue strength of a welded joint can be as little as 20 percent of the unwelded plate (see, e.g. [12]). A more sophisticated fatigue crack-growth analysis that includes the effects of corrosion as well as load interactions and occasional impulsive loads may be incorporated into the future study using the Monte Carlo technique. In addition, a Monte Carlo technique may be used to obtain insight into the influence of the uncertainties associated with the predicted sea loads on the propagation life. The LEFM used in this study is one of the two methods for dealing with fatigues. Thus Monte Carlo studies of the Palmgren-Miner and Forman models, of load-sequence effects, and of the influence of residual strains are also suggested.

Table 1. Average* values of yield strength (σ_y) and ultimate strength (σ_u) of ASTM A517 Grade F steel and CSA G40.21 Grade 350WT at strain rate 0.0001 s^{-1} (after [6]).

Steel	Temp. (°C)	σ_y (MPa)	σ_u (MPa)
A517	0	777	834
	-40	804	869
350WT	0	452	539
	-40	470	560

*Based on three tensile tests

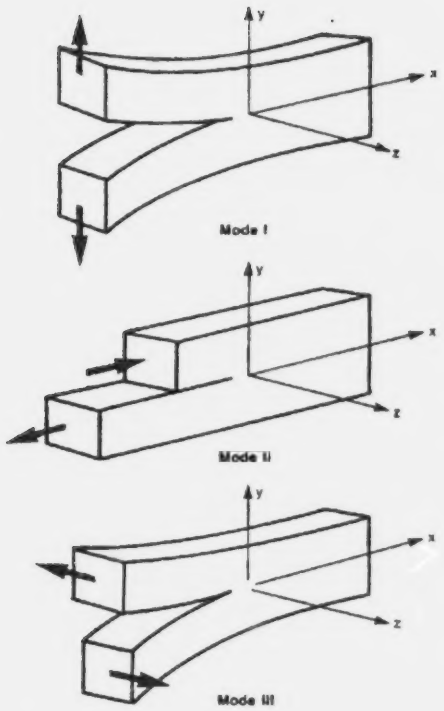


Figure 1. Modes of crack surface displacement

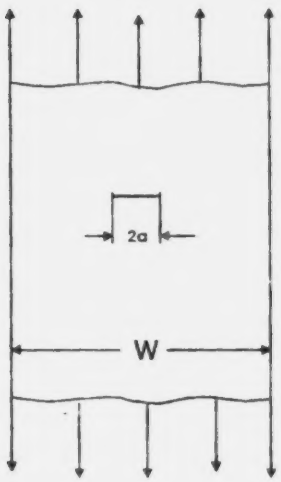
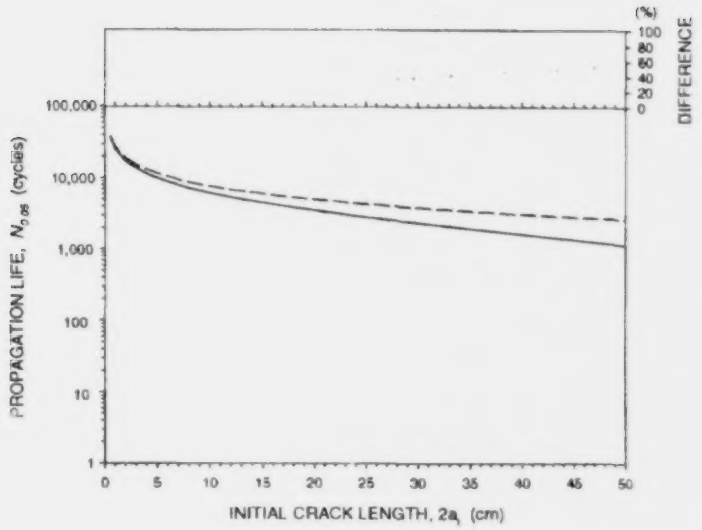


Figure 2. Crack of length $2a$ in plate of width W under random loading. Here $W \gg 2a$ by assumption

P516987.PDF [Page: 28 of 39]

(a) b = 50 MPa



(b) b = 70 MPa

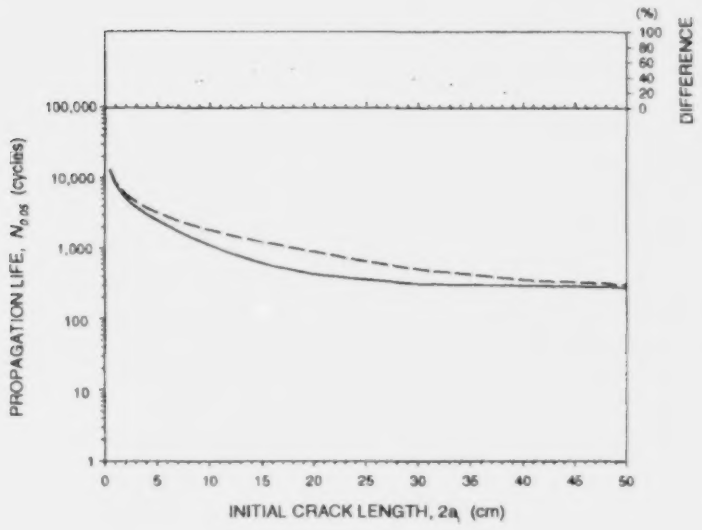
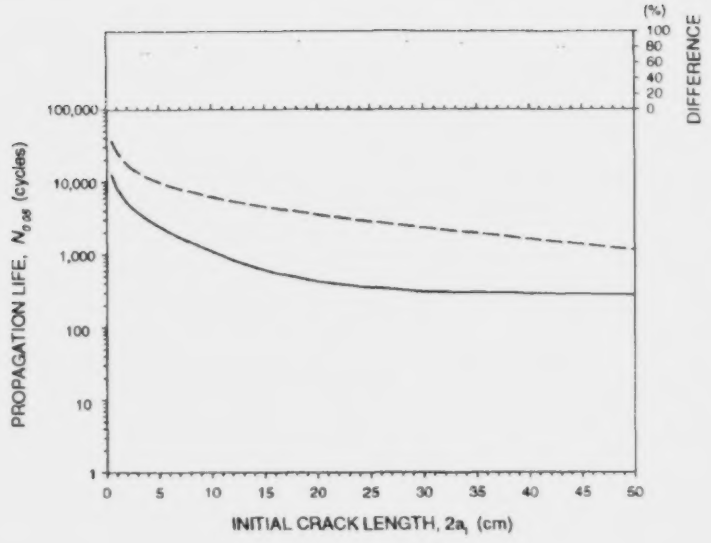


Figure 3. Fifth percentile $N_{0.05}$ of simulated propagation life N versus initial crack length (i.e., probability that the number of load cycles for a crack of initial length $2a_i$ cm to reach critical length given by equation (13) in less than N cycles is 0.05) when σ_{max} is Rayleigh distributed with (a) mode $b = 50$ MPa and (b) mode $b = 70$ MPa. **Legend**: ——— K_{I_c} = 100 MPa$\sqrt{m}$, - - - - K_{I_c} = 150 MPa$\sqrt{m}$, and ····· percent difference (decrease) between Ns for the two values of K_{I_c} relative to N corresponding to K_c = 150 MPa$\sqrt{m}$.

P516987.PDF [Page: 29 of 39]

(a) K_{I_c} = 100 MPa$\sqrt{m}$



(b) K_{I_c} = 150 MPa$\sqrt{m}$

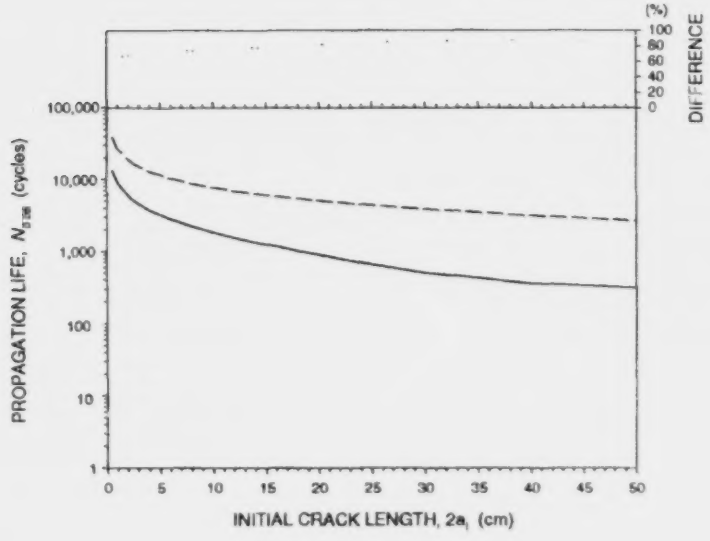


Figure 4. Fifth percentile $N_{0.05}$ of simulated N versus initial crack length when (a) K_{I_c} = 100 MPa$\sqrt{m}$, and (b) K_{I_c} = 150 MPa$\sqrt{m}$. The critical length is given by equation (13). **Legend:** ——— σ_{max} is Rayleigh distributed with mode b = 70 MPa, – – – – with mode b = 50 MPa, and ····· percent difference (decrease) between Ns for the two values of b relative to N corresponding to b = 50 MPa.

P516987.PDF [Page: 30 of 39]

References

1. Meguid, SA, *Engineering Fracture Mechanics*, Elsevier, London, 1989.
2. Rolfe, ST and Barsom, JM, *Fracture and Fatigue Control in Structures*, Prentice-Hall, Englewood, New Jersey, 1977.
3. Rubinstein, RY, *Simulation and the Monte Carlo Method*, Wiley, New York, 1981.
4. Forman, RG, Kearney, VE, and Engle, RM, "Numerical analysis of crack propagation in cyclic-loaded structures", *J. Basic Engng*, v.89, 1967, pp.459-464.
5. Walker, K. in *Effects of environment and complex load history on fatigue life*, ASTM STP 462, American Society for Testing and Materials, Philadelphia, 1970, pp.1-14.
6. Tiku, S, "Material Property Database Overview", Fleet Technology Ltd., Kanata, Ontario, Contract W8477-6-RQ02/A, December 1999, PWGSC.
7. Chang, JB, Szamossi, M, and Liu, K.-W., "Random Spectrum Fatigue Crack Life Predictions with or without Considering Load Interactions", in *Methods and Models for Predicting Fatigue Crack Growth under Random Loading*, ASTM STP 748, American Society for Testing and Materials, Philadelphia, Pa, 1981, pp. 115-132.
8. Bransom, JM, "Fatigue crack growth under variable amplitude loading", in *Progress in Flow Growth and Fracture Toughness Testing*, STP 536, ASTM, 1973.
9. Rudd, JL and Engle, RM, Jr., "Crack growth behavior of center-cracked panels under random spectrum loading", in Chang, JB, CM Hudson (eds), *Methods And Models for Predicting Fatigue Crack Growth Under Random Loading*, ASTM Special Technical Publication 748, American Society for Testing and Materials, Philadelphia, Pa, 1981.
10. Madsen, HO, Krenk, S, and Lind, NC, *Methods of Structural Safety*, Prentice-Hall, Englewood Cliff, NJ, 1986.
11. MaKay, MD, Beckman, RJ, and Conover, WJ, "A comparison of three methods for selecting values of input variables in the analysis of output from a computer code", *Technometrics*, v.21, No.2, May 1979, pp. 239-245.
12. Mann, JY, *Fatigue of Materials*, Melbourne University Press, Melbourne, Australia, 1967.
13. Pock, LP and Smith, RA in *Fracture Mechanics: Current Status, Future Prospect*, Proceedings of a conference held at Cambridge University, 16 Mar 1979, RA Smith (ed.), pp.29-67.

P516987.PDF [Page: 31 of 39]

Annex A: Effects of stress ratio *R*

Effect on fatigue initiation threshold

If the value of the stress-intensity factor is less than the threshold value ΔK_{th}, fatigue crack growth does not occur no matter how many times the cyclic loads occur. The value of ΔK_{th} is dependent on the stress ratio R such that the greater the value of R, the lower the value of ΔK_{th}. As well, the value of ΔK_{th} is found to depend on the mean stress intensity. For mild steel of ultimate tensile strength of 430 MPa, for example, the following values are given in Pock and Smith [13]:

$R = \sigma_{min} / \sigma_{max}$	ΔK_{th} (MPa$\sqrt{m}$)
-1	6.4
0.49	4.3
0.75	3.8

Fatigue-crack growth rate models based on *R*-ratio

There are fatigue-crack growth-rate models that modify the Paris law (3) explicitly to take the values of R-ratios into account. Such expressions include one due to Forman et al. [4]:

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_c - \Delta K} \tag{A.1}$$

where K_{I_c} is the fracture toughness, or the critical stress intensity factor. Forman's equation is valid only for $R > 0$. As such, it is not appropriate for this study, since as the ship traverses in waves, the hull girder will usually experience alternating sagging and hogging stresses, so $R < 0$. Another expression is due to Walker [5]:

$$\frac{da}{dN} = \begin{cases} C\left[\frac{\Delta K}{(1-\bar{R})^{1-n}}\right]^m & \text{for } \Delta K > \Delta K_{th} \\ 0 & \text{for } \Delta K < \Delta K_{th} \end{cases} \tag{A.2}$$

with

$$\bar{R} = R \quad \text{for } 0 \le \bar{R} \le R^+_{cut},$$
$$\bar{R} = R^+_{cut} \quad \text{for } 0 \le \bar{R} \ge R^+_{cut},$$

where C and m are the fatigue crack growth rate constants, R is the stress ratio (8), n is the Walker stress ratio layer "collapsing" factor, and R^+_{cut} is the cutoff value of the stress ratio above which no further stress-ratio layering is shown in a da/dN versus ΔK plot. Because the exponents m and n can only be determined by fitting the experimental data, this equation is not appropriate for this study.

Annex B: Probability distributions

(1) **Lognormal** (μ,σ) represents a lognormal distribution with mean values μ and standard deviation σ :

$$f(x) = \frac{1}{x\sqrt{2\pi}\sigma_l} \exp\left\{-\frac{(\ln x - \mu_l)^2}{2\sigma_l^2}\right\} \tag{B.1}$$

where

$$\mu_l = \ln\left(\frac{\mu^2}{\sqrt{\sigma^2+\mu^2}}\right), \quad \sigma_l = \sqrt{\ln\left(\frac{\sigma^2+\mu^2}{\mu^2}\right)}$$

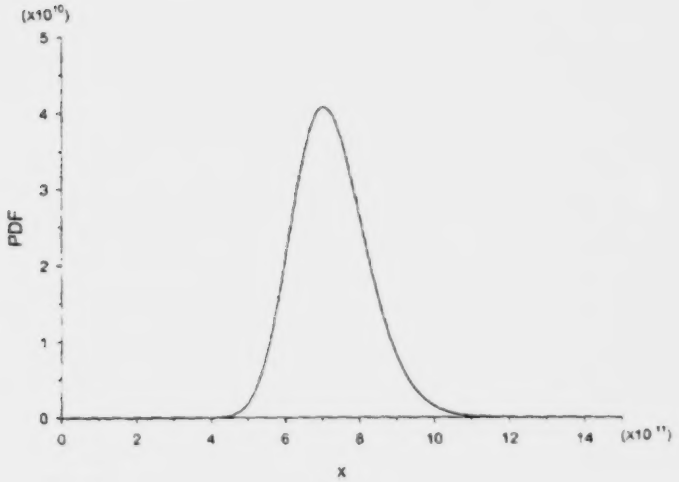


Figure B1. Lognormal distribution $(\mu, \sigma) = (7.23\text{x}10^{-11}, 1.0\text{x}10^{-11})$

(2) **Truncated-Normal** $(\mu,\sigma,x_{min},x_{max})$ represents a truncated normal distribution for x between x_{min} and x_{max} with mean values μ and standard deviation σ .

$$f(x) = \frac{\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}}{\Phi(x_{min}) - \Phi(x_{max})}, \tag{B.2}$$

where $\Phi(x)$ is the cdf for the normal distribution.

P516987.PDF [Page: 33 of 39]

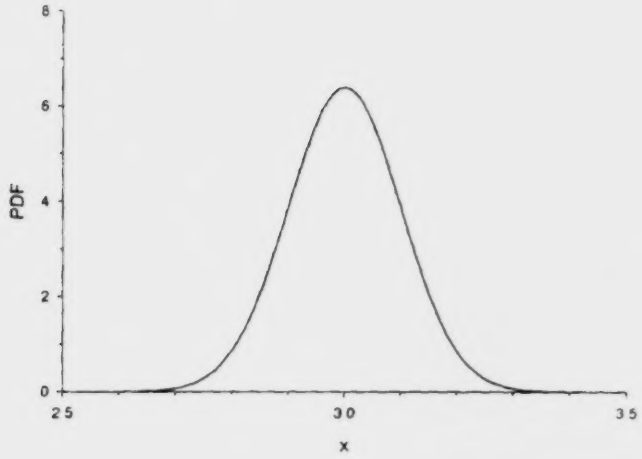


Figure B2. Truncated-Normal distribution. $(\mu, \sigma, x_{min}, x_{max}) = (3.0, 0.1, 2.85, 3.05)$

(3) **Rayleigh (*b*)** represents a Rayleigh distribution with mode *b*:

$$f(x) = \frac{x}{b^2}\exp\left(-\frac{x^2}{2b^2}\right) \tag{B 3}$$

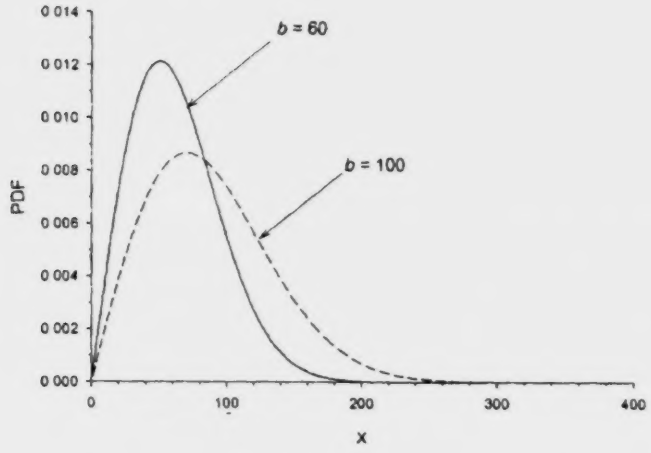


Figure B3. Rayleigh distribution: $b = 60, 100$

Annex C: Relative frequency histogram for propagation life

As an example, Fig. C1 shows the relative frequency of $N_{0.05}$, the 5th percentile of the simulated propagation life N (i.e. the probability that the number of load cycles for a newly-discovered crack to reach a critical length in less than N is 0.05). The histogram is obtained from a set of 10^4 simulated N's for the initial crack length $2a_i = 10$ cm when σ_{max} is Rayleigh distributed with mode $b = 50$ MPa ($\Delta\sigma = \sqrt{2}\sigma_{max}$) and $K_{I_c} = 150$ MPa$\sqrt{m}$. In all 10,000 simulations, the range of the N spans many orders of magnitude. For example, in the particular set shown here, the minimum and maximum values are 360 and 3.58×10^{10} cycles, respectively. Thus, the bin size in the figure is so chosen that bin(n+1)= $10^{0.2}\times$ bin(n).

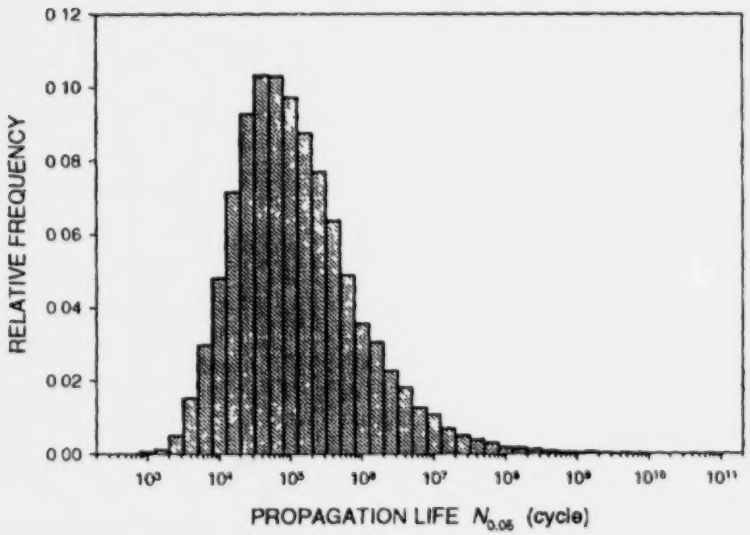


Figure C1. Relative frequency of $N_{0.05}$ from 1000 samples for a crack of initial length of 10 cm when σ_{max} is Rayleigh distributed with mode $b = 50$ MPa and $K_{I_c} = 150$ MPa$\sqrt{m}$.

P516987 PDF [Page: 35 of 39]

Annex D: Variations among realizations for identical initial conditions

The variables and parameters in the Monte Carlo method are simulated by computer-generated random numbers possessing the prescribed probabilities. The crack propagation life shown in Figs. 3 and 4 represent the 5th percentile of the simulated 10,000 N versus the initial crack length. For each set, different random numbers are generated. So if multiple sets of simulations are conducted for identical conditions, resulting histograms such as shown in Annex D are similar in shape but may be different in details. The figure below shows a typical case of such variations in the predicted propagation life from three sets of outcomes. In this particular case, the random variations are seen to be within about 7 percent (relative to the largest value).

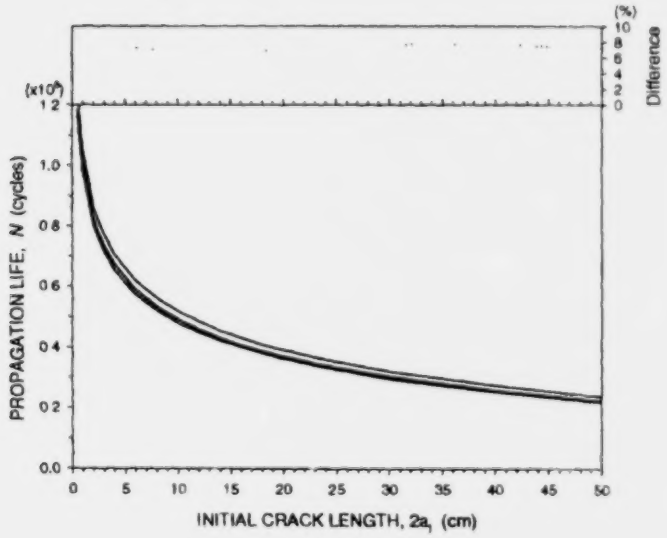


Figure D1. Propagation life versus initial crack length (i.e., probability that number of load cycles for a newly-discovered crack to reach critical length is no less than N is 0.95) when σ_{max} and $\Delta\sigma$ are Rayleigh distributed with mode $b = 60$ MPa and $K_{I_c} = 300$ MPa$\sqrt{m}$. The broken line shows percent difference (decrease) between Ns for the largest and smallest values of K_{I_c} relative to N corresponding to the largest for a given initial crack length.

P516987.PDF [Page: 36 of 39]

List of symbols/abbreviations/acronyms/initialisms

DND	Department of National Defence
ISSMM	Improved Ship Structural Maintenance and Management program
LEFM	linear elastic fracture mechanics
a	crack size
da/dN	cyclic crack growth rate
a_i	initial crack size
a_c	critical crack size
C	experimentally determined constant in Paris equation
F	cumulative distribution function
f	probability density function
K	stress intensity factor
K_{I_c}	fracture toughness in mode-I (critical stress intensity factor of mode I)
K_{min}, K_{max}	minimum and maximum stress intensity factors
ΔK	stress-intensity-factor range
ΔK_{eff}	effective stress-intensity-factor range
m	material constant in Paris equation
m_n	nth spectral moment
N	number of cycles
$N_{0.05}$	5-percentile of N
p	uniformly distributed number between 0 and 1
Q	constant
R	stress ratio
rms	root mean square
$\bar{T}_0$	average up-crossing period
σ	stress or standard deviation
σ_{min}, σ_{max}	minimum and maximum applied stresses
$\Delta\sigma_{eff}$	effective applied stress range
μ	mean value
Φ	cumulative distribution function for normal distribution

P516987.PDF [Page: 37 of 39]

UNCLASSIFIED
SECURITY CLASSIFICATION OF FORM
(highest classification of Title, Abstract, Keywords)

DOCUMENT CONTROL DATA

(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)

1 ORIGINATOR (the name and address of the organization preparing the document. Organizations for whom the document was prepared, e g Establishment sponsoring a contractor's report, or tasking agency, are entered in section 8)

Defence Research Establishment Atlantic
PO Box 1012
Dartmouth, Nova Scotia, Canada B2Y 3Z7

2 SECURITY CLASSIFICATION (overall security classification of the document including special warning terms if applicable)

UNCLASSIFIED

3 TITLE (the complete document title as indicated on the title page Its classification should be indicated by the appropriate abbreviation (S,C,R or U) in parentheses after the title)

Monte Carlo Simulation of Crack Growth on Ship Hull Plating

4 AUTHORS (Last name, first name, middle initial If military, show rank, e g Doe, Maj John E)

Samon Ando

5 DATE OF PUBLICATION (month and year of publication of document)	6a NO OF PAGES (total containing information Include Annexes, Appendices, etc)	6b NO OF REFS (total cited in document)
October 2001	23	13

7 DESCRIPTIVE NOTES (the category of the document, e g technical report, technical note or memorandum. If appropriate, enter the type of report, e g interim, progress, summary, annual or final Give the inclusive dates when a specific reporting period is covered)

TECHNICAL MEMORANDUM

8 SPONSORING ACTIVITY (the name of the department project office or laboratory sponsoring the research and development Include address)

Defence Research Establishment Atlantic

9a PROJECT OR GRANT NO (if appropriate, the applicable research and development project or grant number under which the document was written Please specify whether project or grant)	9b CONTRACT NO (if appropriate, the applicable number under which the document was written)
1gb13	

10a ORIGINATOR'S DOCUMENT NUMBER (the official document number by which the document is identified by the originating activity This number must be unique to this document)	10b OTHER DOCUMENT NOs (Any other numbers which may be assigned this document either by the originator or by the sponsor)
DREA TM 2001-166	

11 DOCUMENT AVAILABILITY (any limitations on further dissemination of the document, other than those imposed by security classification)

(X) Unlimited distribution
( ) Defence departments and defence contractors; further distribution only as approved
( ) Defence departments and Canadian defence contractors; further distribution only as approved
( ) Government departments and agencies, further distribution only as approved
( ) Defence departments; further distribution only as approved
( ) Other (please specify):

12 DOCUMENT ANNOUNCEMENT (any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availability (11) However, where further distribution (beyond the audience specified in (11) is possible, a wider announcement audience may be selected)

UNCLASSIFIED

SECURITY CLASSIFICATION OF FORM
UNCLASSIFIED
SECURITY CLASSIFICATION OF FORM
(highest classification of Title, Abstract, Keywords)

13. ABSTRACT (a brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), (R), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual)

A Monte Carlo study of the propagation lives of through-thickness cracks in ship's deck plate subjected to mode-I deformation and their sensitivity to various parameters was carried out. The simulation model is based on the Paris law, and takes into account those uncertainties that affect demand (stress-intensity factor) and capability (fracture toughness). The effects of load sequence on the crack-growth rate were ignored, and the random loading on the hull girder was approximated by equivalent constant-amplitude loading. The results are presented in plots of the statistical estimate of the propagation life versus the initial crack length. The propagation life was seen to be relatively insensitive to the initial crack length when the latter was greater than about 10 cm but sensitive to the applied stress range.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus, e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus-identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title)

fatigue
crack propagation
computer simulation
Monte Carlo method
ship hull

UNCLASSIFIED
SECURITY CLASSIFICATION OF FORM

CLASSIFICATION / DESIGNATION
WARNING TERM

A mod. 17 Dec 1997